DETERMINATION OF MOVEMENT STABILITY OF ESPECIALLY LARGE CLASS HYBRID BUS WITH ACTIVE TRAILER

/ ДО ВИЗНАЧЕННЯ СТІЙКОСТІ РУХУ ГІБРИДНОГО АВТОБУСА ОСОБЛИВО ВЕЛИКОГО КЛАСУ З АКТИВНИМ ПРИЧЕПОМ

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ABSTRACT
The paper aims to achieve the normalized maneuverability indicators for buses of especially large class (hinge-jointed) possible by appropriate layout schemes and use of controlled (self-installed) wheels of trailer's section (trailer) which drive can be carried by an electric motor located in the trailing section. Furthermore, the electric motor may be pulling for the bus during movement in urban "traffic jams" when movement speed does not exceed 5 m/s.
In view of the lateral wheel's abduction of SHZA was determined total resistance power for rolling wheels. In addition there was defined electric motor power that is set in the trailing section of the bus, based on the need to perform the following modes of movement as starting out from place, rectilinear bus motion, motion along a circle and turn the wheels of trailer's section to improve the maneuverability of an articulated bus.
Analysis of static stability conditions SHZA at the implementation of tractive force on the axle of the trailer showed that the coefficient of aerodynamic resistance will not affect the value of the critical velocity (not included in the expression of critical speed), and the coefficients of rolling resistance on the first and second axes slightly affect the value of the critical velocity and a critical value of parameter alpha, which determines the amount of tractive effort. However, even under by optimum selected stiffness and selected layout parameters critical speed of the hinge-articulated buses does not exceed 12.5 m/s, those buses requiring special system against assembly.

INTRODUCTION
Transportation is one of branched industries that provide movement of people. The social importance of transport is reduced to increase efficiency and labor productivity of citizens due to reductions in the transport fatigue of daily trips (productivity drops to 10-15% if the time of trip exceeds 40 minutes, and even more if the waiting time of transport is more than 15 minutes). The development of transport speeds population displacement, improve cultural level and the public mood (Smorodintseva E.E., 2013).
Recently, on the streets of cities it has observed a deterioration in passenger traffic, intensive motorization leading to a sharp decrease in the speed of public transport. Public transport, on average, is moving at a speed less than 30 km/h (Sakhno V.P., 2011). The reason - traffic jams.

According to the information of International Union of Public Transport, urban public transport requires at the same carrying capacity in 20 times smaller the area of the road network compared with individual cars. Modern bus in 5 times less polluting the atmosphere and requires 3 times less energy costs per one transported passenger compared with individual passenger car (Balabaeva l., 2008).

According to recommended (based on national studies) norms the ratio between the number of buses of various classes, which are used in big cities, three-quarters of the entire bus fleet of city with population over 1 million residents should be buses of large and especially large class (45% and 30% respectively). The feasibility of such a ratio is confirmed by foreign practices: at the disposal of Autonomous Management of Paris Public transport (RATP) to 21% of buses especially large class (Belevtsova N.L., 2014).

Uncontrolled replacement of buses of large and especially large classes to minibuses resulted in worsening traffic situation in the city streets; bus congestion landing sites; increasing the probability of traffic accidents; increase emissions of toxic substances into the environment.

For any vehicle, including for the bus, the main purpose parameters (indicators of its ability to perform its functions) are dimensions, mass settings, speed and dynamic characteristics of the performed transport work and others. Depending on the service conditions (traffic and transport) to the fore go various settings. For all urban buses, these settings are passenger capacity, rate of passenger exchange, acceleration dynamics, stability, manageability, and for especially large urban, besides maneuverability. In most countries the overall length of single buses is limited to 12 meters, although there are structures up to 15 meters and articulated ones of 18 m. This is explained by the need to meet the requirements of the prescriptions of UNECE №36 and GOST 27 815 - 88, particularly p.p.5.9.1. «... when in motion on the turn on right as on left, when most protruding body points or bumper in circle radius of 12.5 m»; and 5.9.2. «... when in motion on the turn on right as on left, when most protruding body points or bumper describe a circle with a radius of 12.5 m, bus should be placed in a corridor 7.2 m» (Regulations ISO UN ECE R 36-03:2002).

Achieving standardized indicators of buses maneuverability especially large class (hinge-jointed) may be possible by appropriate layout schemes and application managed (bearing) wheels towed section (trailer), which drive can be carried by an electric motor located in the trailing section. Furthermore, this electric motor may be pulling for the bus when driving in urban "traffic jams", when speed is no less than 5 m/s. Therefore, the work’s purpose is determination the parameters of sustainability of the bus when the driving element is a trailer section.

MATERIAL AND METHOD

In the article (Sakhno V.P., 2015) is proposed to determine the power of electrical motor of hybrid auto train based on the condition to provide movement of road train with a minimum speed in conditions of road train maneuvering and opportunities for movement in urban areas with "creeping" speed.

Road train’s maneuvering is connected with its movement along the trajectory of the great curvature. The decisive here is the dimensional road train lane that is defined by its inner and outer dimensional radiuses. These radiuses can be determined or experimentally, or by means of calculations with the help of mathematical model.

In the article (Sakhno V.P., 2011) presented a system of equations, describing motion of road train by circle:

\[
\begin{align*}
(m+m_j)U-(cm_1+m_i+d_i)\cos \varphi_1+m_i+d_i \times \cos \varphi_1 \dot{\varphi}_1+(m+m_j)\omega V-m_j \dot{\varphi}_j + \sin \varphi_1 - \\
= Y_1 \cos \varphi_1 - X_1 \sin \varphi_1 + Y_1 \cos \varphi_1 - X_1 \sin \varphi_1 + \Sigma (Y_j + Y_j') + \Sigma (Y_j + Y_j');
\end{align*}
\]

\[
-c_m U + \left(c'm_1 + c'm_i + d_1 \cos \varphi_1 - c_m + c'm_i + d_1 \sin \varphi_1, \right) + \Sigma (Y_j + Y_j') + \Sigma (Y_j + Y_j');
\]

\[
(m+m_j)U-(cm_1+m_i+d_i)\cos \varphi_1+m_i+d_i \times \cos \varphi_1 \dot{\varphi}_1+(m+m_j)\omega V-m_j \dot{\varphi}_j + \sin \varphi_1 - \\
= (X \sin \varphi_j - Y \cos \varphi_j) (c \sin \varphi_j + a)+(Y \cos \varphi_j - X \sin \varphi_j) (a + c \sin \varphi_j) + (Y \sin \varphi_j + X \cos \varphi_j) \times
\]

\[
(X \cos \varphi_j - Y \sin \varphi_j) (c \sin \varphi_j + a)+(Y \cos \varphi_j - X \sin \varphi_j) (a + c \sin \varphi_j) + (X \sin \varphi_j + X \cos \varphi_j) \times
\]

\[
Y \sin \varphi_j + X \cos \varphi_j) - \Sigma (Y_j + Y_j') - M_j - \Sigma (Y_j + Y_j');
\]

\[
= \Sigma (Y_j + Y_j') + M_j;
\]

\[
= \Sigma (Y_j + Y_j') + M_j + M_j;
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= \Sigma (Y_j + Y_j') + M_j + M_j;
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= \Sigma (Y_j + Y_j') + M_j + M_j;
\]
In the above described system of equations the following notations were made: 
m, m₁, l, l₁ – respectively the mass and moment of inertia of car-tractor and semitrailer; 
v, u, ω - longitudinal, lateral and angular speed of the tractor; 
a, b, c, d₁, l₁, ε - auto train layout parameters; 
X₁, Y₁, X′₁, Y′₁, X_j, Y_j, X_j′, Y_j′, X_{j2}, Y_{j2}, X_j', Y_j' – respectively longitudinal and lateral forces on the 
wheels of front axle of the tractor, rear axles of tractor and axles of the semitrailer (i=2, j=3); 
θ, φ – rotation angle of the tractor’s steering wheels and assembly of auto train. 
Expressions for the longitudinal and lateral velocity of mass center for semitrailer are written in the form: 
v₂ = v₁ \cos φ₁ - (u₁ - a₁ (b₁ - c₁)) \sin φ₁, 
\quad u₂ = v₁ \sin φ₁ + (u₁ - a₁ (b₁ - c₁)) \cos φ₁ - a₁ a₁. 
(2)

Angular velocity of semitrailer will be equal to angular velocity of road tractor: 
\ω₂ = ω₁ = const.

Considering the fact that abduction angles of trailer’s axes do not exceed 10°, tangents of these 
angles can be considered equal values to angles with an error that does not exceed 1%. Therefore, the 
expressions for defining angles of abduction are written in the form: 
\δ₁ = θ₁ - \frac{u₁ + a₁ b₁}{v₁}; \quad \delta₂ = \frac{u₁ + a₁ b₁}{v₁}; \quad \delta₃ = \frac{u₂ + a₂ b₂}{v₂}; \quad \delta₄ = \frac{u₂ + a₂ b₂}{v₂}; \quad \delta₅ = \frac{u₂ + a₂ b₂}{v₂}; \quad \delta₆ = \frac{u₂ + a₂ b₂}{v₂};
(3)

After determining the longitudinal and lateral velocity of bus and trailers, section angles abduction 
follows: θ₁= 0.051 rad., θ₂= 0.048 rad., θ₃= 0.044 rad., θ₄= 0.045 rad., θ₅= 0.045 rad.

Considering the lateral abduction of wheels SHZA, the total power of resistance for bus’s wheels 
rolling is defined as: 
\mathbf{P_f} = \Sigma G_i x g \times f_{cyu},
(4)

where \( f_{cyu} \) - coefficient of rolling resistance caused by the interaction of wheels with a support surface and 
lateral diverting wheel; 
Gᵢ – weight that accounted on the axis of the bus and trailer’s section, \( \Sigma G_i = 28000 \) kg.

At rolling wheel with abduction, except of radial deformation of tire it also deforms in the lateral 
direction that increases losses on rolling. The coefficient value of rolling resistance by extraction can be 
determined by empirical formulas. Formula of Professor V.A. Illarionov is as follows:
\[ f_{rr} = f_0 + \frac{k_{rr} \delta^2_{rr}}{F_Z} \]
(5)

where \( f_{rr}, f_0 \) - coefficient of rolling resistance wheels with and without taking into account the abduction of wheels, \( f_0 = 0.015; \)
\( \delta_{rr} \) - abduction angle of wheels axes of auto train, rad.; 
\( k_{rr} \) - resistance coefficient of lateral abduction of wheels axes of auto train, 
\( F_Z \) - load on the wheel of auto train.

Due to the fact that the angle of auto train lateral axis's abduction is different, like as the resistance 
coefficient of lateral abstraction of wheel’s axes of SHZA, then the total coefficient of rolling resistance and 
rolling resistance force are determined for each individual axis of SHZA. In view of this fact, rolling resistance 
force of auto train was \( P_f = 6311 \) N at an average coefficient of rolling resistance \( f_{rr}=0.023. \)

The power of electric motor based on the need to perform the following modes of motion was defined.

1) Starting from the point. In this mode, the aim is to overcome rolling resistance of bus’s wheels and 
bus force of inertia that is counted as additional rolling resistance.

Then: 
\[ P_f = G_{an} \times g \times f = 28000 \times 9.8 \times 0.03 = 8232 \text{ N}, \]
where \( G_{an} \) – gross bus’s weight, \( G_{an} = 28000 \) kg;
f – coefficient of rolling resistance at bus starting, \( f = 0.03 \).
g – acceleration of free falling, \( g = 9.8 \text{ m/s}^2 \).

The force of bus inertia:

\[
P_j = \delta \times G_{an} \times j = 3.5 \times 28000 \times 0.3 = 9800 \text{ N},
\]

where
\( \delta \) – coefficient that takes into account growth of inertia forces translational mass of bus by rotating masses, \( \delta = 3.5 \text{ Ns}^2/\text{m}^4 \) (Smorodintseva E.E., (2013));
\( j \) – acceleration at bus start, \( j = 0.1 \text{ m/s}^2 \).

Thus, resistance force of movement at bus start will be:

\[
P_{on} = P_j + P_p = 18032 \text{ N}
\]

The traction force that is possible for implementation on axis of the trailers section:

\[
P_{PT} = G_p \cdot \varphi = 10000 \times 9.8 \times 0.6 = 58860 \text{ N}
\]

Since the traction force that is possible for implementation on wheels of trailers section SHZA, is more than the sum of resistance forces of motion, it will provide it with the starting out from the place.

Power required for bus starting out at speed \( v = 1 \text{ m/s} \) will be:

\[
N = \frac{P_{on} \times v}{1000 \times \eta} = 18.98 \text{ kW}
\]

where
\( \eta \) - coefficient of transmission efficiency in the transmission of power from electric motor to the driving wheels.

Power required for straight-line motion of bus at speed \( v = 1...2 \text{ m/s} \) will be:

\[
N = \frac{P_j \times v}{1000 \times \eta} = 4.34...8.68 \text{ kW}
\]

Power required for auto train’s motion around the circle at speeds 1...2 m/s will be:

\[
N_r = \frac{P_{pr} \times v}{1000 \times \eta} = 6.64...13.28 \text{ kW}
\]

Therefore, required power of electric motor for the bus rectilinear movement at a speed of 1.0...2.0 m/s, generally does not exceed 9 kW, while at starting from the point is - 19 kW. According to motor power 19 kW bus can move along rectilinear area with a maximum speed of 4.4 m/s (16 km/h).

If the electric motor that is located on the trailing section is used for turning the wheels, it is necessary to identify the electric motor power to rotate the steering wheels of the axle.

In the basis of selection and justification of drive management over the wheels of the semitrailer should put reliance of moment resistance turning his steering wheels from constructive and operating factors.

The most complete method of determining the points of resistance at turning the steering wheels of the car-tractor and semitrailer when driving auto train was developed by A.P. Soltusom in his work (Sakhno V.P., 2015). According to this method in the work (Sakhno V.P., 2015) was determined resistance turning point of trailer driven axles for its load within 80 kN. For SHZA load on axle of trailer section is typically 100 kN. Therefore, we define the moment of resistance rotation wheels of axis according to the method (Sakhno V.P., 2015). According to this work, force interaction of driven wheels with supporting surface in motion should be considered for three cases:

- rectilinear motion (in practice rectilinear motion of the vehicle, and therefore of controlled wheel carried out by coupling curves of large radius);
- move on a curved trajectory of constant curvature;
- move on a curved trajectory of variable curvature.

Special interest represents motion dynamics of elastic controlled wheel on a curved trajectory of variable curvature, as the first two of them can be viewed (in terms of force interaction of elastic wheel to the supporting surface) as partial cases.
At the motion of controlled elastic wheel on a curved trajectory of variable curvature are affected the force of gravity, inertial and side forces; forces due to uncontrollable car wheels that moving curvilinear trajectory; the resistance movement and rotation about the axis of kingpin. In general, the reaction motion in the tire footprint with the supporting surface is reduced to three resultants, applied at the center of a print, and points in respect of each axis of coordinates. And this is correct in the absence of rotation of controlled wheel and for the axis of kingpin. In the presence of the angular velocity of rotation, pins of driven wheels on axle kingpin have additional resistance turning point, due to the angular velocity of rotation pins.

Due to the structural parameters of controlled bridge, points of the resultant reactions bearing surface that brought to the center of a print tires, are shifted as for the axis of the kingpin.

As the result of this shift, each of resultant creates relatively the axis pivot point. It is obvious that for a turn of managed the wheel concerning kingpin axis, it is necessary to overcome these moments. Consider each of them.

The moment of resistance to rotation of the controlled wheels of the car-tractor and semitrailer at the motion of auto train (with sufficient accuracy for practical calculations) can be represented as such (Sakhno V.P., 2015):

\[
M(\ddot{\omega}) = \sum M_a(\ddot{\omega}) + M_{Ru}(\ddot{\omega}) + M_{Rd}(\ddot{\omega}) + M_{Tc}(\ddot{\omega}) + M_{Ts}(\ddot{\omega}) + M_{Rw}(\ddot{\omega})
\]

where \(M_a(\ddot{\omega})\) - moment of resistance to rotation of the steering wheels relative to the axis kingpin; \(M_{Ru}(\ddot{\omega})\), \(M_{Rd}(\ddot{\omega})\), \(M_{Rw}(\ddot{\omega})\) - components of resistance rotation moment, which caused in accordance with angular velocity of rotation pins, stabilizing tire’s moment that arises in the result of rolling of steering wheels with abduction; weighted stabilizing factor; moments caused by resultant lateral and longitudinal reactions of supporting surface on the steered wheels, and friction in node.

Calculation of points resistance of rotation of the steering wheels of trailer section performs on condition the mass that falls on its wheels is 9,000 kg.

Weighing stabilizing moment at the combined inclination pivot axis defined by the following dependencies (Sakhno V.P., 2015):

- for left steering wheel:
  \[M_{R1} = M_{Rzuo1} + M_{Rzuw1}\]

- for right steering wheel:
  \[M_{R2} = M_{Rzuo2} + M_{Rzuw2}\]

where \(M_{Rzuo1,2}, M_{Rzuw1,2}\) - weight stabilizing moments caused in accordance longitudinal and transverse inclination of axis pivot \(\alpha_{uo}, \beta_{uo}\)

At the mass on steering axle of trailer’s sections 9000 kg, weighing stabilizing point on the left wheel amounted in 776.84 Nm, on the right – 779.58 Nm, when calculating weighing stabilizing moment can be limited to just one wheel and double the result.

The resulting moment due to combined lateral reactions at combined inclination of kingpin is determined by dependency:

- for left steering wheel:
  \[M_{R1} = R_{\alpha_1} \sin(\alpha_{uo} \cos(\gamma_{uo} \sin(\theta_{uo} + \theta_{uo} - \gamma_{uo}) + \cos(\gamma_{uo} \sin(\theta_{uo} + \theta_{uo} - \gamma_{uo})) + R_{\alpha_2} \cos(\gamma_{uo} \sin(\alpha_{uo} \cos(\gamma_{uo} \sin(\theta_{uo} + \theta_{uo} - \gamma_{uo})) + \cos(\gamma_{uo} \sin(\theta_{uo} + \theta_{uo} - \gamma_{uo})\]

- for right steering wheel:
  \[M_{R2} = R_{\alpha_1} \sin(\alpha_{uo} \cos(\gamma_{uo} \sin(\theta_{uo} + \theta_{uo} - \gamma_{uo}) + \cos(\gamma_{uo} \sin(\theta_{uo} + \theta_{uo} - \gamma_{uo})) + R_{\alpha_2} \cos(\gamma_{uo} \sin(\alpha_{uo} \cos(\gamma_{uo} \sin(\theta_{uo} + \theta_{uo} - \gamma_{uo})))\]

where \(\theta_{uo}+\gamma_{uo}+\gamma_{uo}(1-\cos\theta_{uo})\) – current angle of collapse.

In contrast to the weighted stabilizing moment, the moment caused by resultant lateral reactions at combined kingpin inclination for left and right wheels significantly differs both in sign and in magnitude. The maximum value of this point is independent of weight that falls on controlled bridge, reaching a peak in the minimum and maximum value of angle between the axis of the kingpin and pins:

- for left steering wheel – 374,1 i 533,5 Nm;
for right steering wheel – (330,2 i 251,3) Nm, 
then, these points should be calculated separately for each wheel.
However, state of this moment in the overall moment of rotation resistance is insignificant and this aspect at engineering calculations can be neglected.
The resulting moment caused by resultant longitudinal reactions at combined inclination of kingpin axis is determined by:
- for left steering wheel:
  \[ M_{Bz1} = R_1 l_s \sin(\theta_1) + \sin(\alpha \omega \sin(\theta_1) \sin(\theta_1 + \theta_1)) = f(Q) \]  
- for right steering wheel:
  \[ M_{Bz2} = R_2 l_s \sin(\theta_2) + \sin(\alpha \omega \sin(\theta_2) \sin(\theta_1 + \theta_2)) = f(Q) \]
Where \( \phi = 0,5 \pi - \alpha \omega \gamma \omega - \angle \) between the axis of pins and pivot.

By calculation’s results, moment of rotation resistance caused by resultant longitudinal reactions from the maximum value of the angle between the axis of kingpin and pin \( \phi = 0,292 \) rad. amounted to 175 Nm, which must be considered when determining the total moment of rotation resistance.
Gyrosopic moment caused by the angular velocity rotation steering wheel around kingpin axis \( \omega_k \) during movement, operates in a plane passing through the axis of the pin and kingpin, causing a redistribution of reaction between the steered wheels of automobiles and is determined by dependence:
\[ M_{\omega} = I_k \omega_k \omega_k \sin(\varphi) \]  
where \( I_k \) – moment of inertia of the wheel concerning an axis of its rotation, \( \omega_k \) – angular velocity of the wheel.
Gyrosopic moment, due to fluctuations in the controlled bridge relatively to the longitudinal axis of the vehicle, which may be caused by irregularities of the supporting surface, kinematics of rotation of the steering wheels in the presence of inclination angles of pins. The value of this moment does not exceed 8 Nm and this moment can be neglected when determining the total moment of rotation’s resistance.
The moment caused by the angular velocity of pin’s rotation if we consider dependence of the resistance moment of the rotation tires \( M_\omega \) of the rotation angleQ, than conditionally function \( M_\omega = f(Q) \) may be divided into three areas (Soltus A.P., 2006):
1. where dependence \( M_\omega = f(Q) \) is linear;
2. where dependence \( M_\omega = f(Q) \) is nonlinear;
3. where \( M_\omega = f(Q) \) is limited traction of tires to the supporting surface and does not depend on the angle of rotation Q.

This conditional division of dependence \( M_\omega = f(Q) \) in three typical areas can significantly simplify, on the one hand, the research of the physical nature of the phenomena, that take place at the interaction of elastic wheels to the supporting surface, and the other - to get comfortable, for practical calculations, dependence determination of the moment \( M_\omega \).

The moment caused by the angular velocity of turnover pin, is defined as:
\[ M_{\omega} = \begin{cases} 
\left( c_\omega \times Q_1 \right) f(Q, Q_A) \\
M_{\omega}^{\text{max}} - \left( c_\omega \times Q_1 \right) \left( \frac{Q_2 - Q_A}{Q_2 - Q_B} \right)^2 f(Q, Q_B) \\
M_{\omega}^{\text{max}} \times f(Q, Q_B) 
\end{cases} \]
where \( M_{\omega}^{\text{max}} \) - limited by the clutch of resistance of moment for rotation;
\( c_\omega \) - angular stiffness of the wheel with tire;
\( Qz, QA, QB \) – corresponding angles at which dependency \( M_\omega = f(Q) \) is linear, nonlinear and limited by tire’s clutch to the supporting surface and does not depend on the value angle of rotation Q.
Considering the fact that the jointed bus works on the road with solid advanced surface, then the moment caused by the angular velocity of pin’s rotation should be determined for the linear dependence \( M_\omega = f(Q) \). The value of this moment does not exceed 12 Nm and this moment can be neglected.
Besides the moment of resistance to rotation, caused by work of CCM during movement, while turning vehicle in pins connection, appears a moment of friction.

Calculation of friction moment in pins node \( M_{\text{fr}} \) brought in general to determine the reactions that act on each bearings of pins, and then - to the direct calculation for known analytical dependencies.
Thus, the magnitude of friction moment in the sleeve is defined by the formula (Sakhno V.P., 2015):

\[ M_{\text{sr}} = \left( \frac{2}{\pi} \right) P_{\text{sr}} f_c d_s \]  

where \( P_{\text{sr}} \) - the force that act on the sleeve, \( f_c \) - coefficient of sliding friction that depends on the material of coupled surfaces of kingpin and bushings, as well as lubricants between them; \( d_s \) - diameter of the sleeve.

The friction moment of heel is defined as:

\[ M_n = \left( \frac{1}{3} \right) P_n f_c \left[ \frac{d_0^2 - d_i^2}{d_0^2 - d_i^2} \right] \]  

where \( P_n \) - the force that acts on the heel; \( f_c \) - coefficient of sliding friction in the heel; \( d_0, d_i \) - respectively outer and inner diameter of the heel.

In the persistent bearing, friction moment is defined as:

\[ M_{\text{norm}} = P_n D f_c \]  

where \( P_n \) - the force that acts on the bearing; \( D \) - diameter of the circle passing through the centers of the balls; \( f_c \) - given friction coefficient of rolling, \( f_c = 0.001 \ldots 0.003 \).

The moment of friction in conical bearings of rolling in general form determined by the dependence:

\[ M_{\text{con}} = R^N d_i \left[ (f_k / d_2) + f_c \sin(\beta/2) \right] \]  

where \( R^N \) - normal to the surface of rolling ball reaction in conical bearing;

\( d_i \) - the average diameter of the track rolling inner ring of bearing;

\( f_k \) - friction coefficient of rolling in conical bearing, \( f_k = 0.001 \);

\( d_2 \) - average diameter of conical surface of roller;

\( f_c \) - coefficient of sliding friction in a pair of friction \('\)roller \(-guide clamp of inner ring of conical bearing,

\( f_c = 0.03 \ldots 0.06 \);

\( \beta \) - the angle between the extreme forming rollers.

For the selected output data moments of friction in the sleeve, the heel, conical rolling bearings and kingpin node \( M_{\text{wst1}} \) made up:

\( M_{\text{em}} = 21.66 \text{ Nm}; \quad M_{\text{p1}} = 13.68 \text{ Nm}; \quad M_{\text{con}} = 22.58 \text{ Nm}; \quad M_{\text{wst1}} = 57.92 \text{ Nm}. \)

For the control wheel module \( M_{\text{Tvl}} = 2M_{\text{Tvl}} = 115.84 \text{ Nm}. \)

Depending on the angle of rotation of the steering wheels, the angular velocity of pin rotation and speed of bus movement in total moment of resistance rotation of wheels varies from 1957 to 1954 Nm.

The moment of viscous friction in steering control hook section is proportional to angular velocity rotation of wheels (Sakhno V.P., 2015):

\[ M_{\text{i}} = h_i \times \dot{\theta}_i \]  

where \( h_i \) - coefficients of viscous friction in the details of steering control, \( h_i = 15 \text{ Nms/rad}; \)

\( \dot{\theta}_i \) - the angular velocity of rotation of steering wheels.

The angular velocity of rotation of steering wheels determined by regime coefficient of rotation \( K_n \), was proposed by Ya.H.Zakinym. This coefficient is determined by the dependence:

\[ K_n = \frac{\dot{\theta}}{\nu}, \]  

where \( \nu \) - the bus speed, m/s.

Regime coefficient of rotation \( K_n \) for real mode of the bus rotation is within 0.1 m\(^{-1}\) for the velocity of the auto-train \( \nu = 5 \text{ m/s}. \) Then, \( \dot{\theta} = 0.5 \text{ rad./s} \) and \( M_i = 7.5 \text{ Nm}. \)

The moment of elasticity in steering control of semitrailer is proportional to angle of rotation of the wheels and stiffness of steering gear (Sakhno V.P., 2015):

\[ M_{\text{p1}} = \chi_1 \times \dot{\theta}_i \]  

where \( \chi_1 \) - stiffness coefficient of the steering gear, \( \chi_1 = 170 \text{ Nm/rad}; \quad M_{\text{p1}} = 85 \text{ Nm}. \)

Thus, the total resistance moment of rotation for the angular velocity of rotation of steering wheels \( \dot{\theta}_i = 0.5 \text{ rad./s} \) made up \( \sum M_i(\theta) = 1967 \ldots 2050 \text{ Nm}. \) Thus the maximum power of the electric motor to rotate
steering wheels of axles of semitrailer does not exceed 1.1 kW, e.g. engine with 20 kW will provide as bus movement with "creeping" speed and the turn of wheels of trailers section.

RESULTS

Model SHZA in the implementation of traction effort on the axis of the trailer gets a range of dynamic properties, specific to the system of "inverted" pendulums. First, note the unfavorable tendency for taking SHZA (case of static stability loss).

Determination of SHZA configurations and other settings of steady state after stability loss of rectilinear motion and the conditions of its stability take the second phase of the study of nonlinear dynamic system. In this formulation of the problem in the model must be enter the equation of longitudinal movement, parameter \( v \) - velocity of longitudinal movement becomes additional variable phase (defined expanded equations and parameter \( \alpha \), which characterizes the tractive force and \( \theta_0 \) – rotation of angle of control wheel module (CCM).

Numerical modeling of system will allow to evaluate the impact of controlled parameter on the dynamic qualities SHZA as a system and to determine corresponding steady state and properties of stability.

When implementing of traction effort on the axis of trailers section SHZA in the equation of planar parallel movement is introduced such a system of parameters (active controlled parameters):

\[
X_1=\alpha Z_3; X_2=\alpha Z_2 - K_{as}v^2; X_3=\alpha Z_3
\]  

where \( kf \) – coefficient of rolling resistance of wheels of the bus axis; \( Z_1, Z_2, Z_3 \) – normal reaction of the bearing surface on the wheels of the bus axis; \( K_{as} \) – factor of streamlining of the bus; \( \alpha \) – factor that determines what portion of the normal reactions of bearing surface is realized in the form of traction force on the driving wheels of trailers links.

The system of differential equations of motion on the phase variables \( (v, u, \omega, \phi, \Phi) \) should be supplemented by a new phase variable - speed of longitudinal movement of mass center of the first level.

Abduction angle of wheels of trailer’s link in expanded form:

\[
\delta_i = -\theta_i - \arctan \left( \frac{v \sin \varphi + (u - \alpha \varphi \cos \omega \varphi ) d_i - (\omega - \varphi ) b_i}{v_i} \right).
\]

\[
v_i = v \cos \varphi - (u - \alpha \varphi ) \sin \varphi.
\]

Characteristic linear dimensions of the first and second level of SHZA:

\[
l = a + b - \lambda, \quad L_i = b_i + d_i
\]

Vertical load on the axis of SHZA and lateral forces of abduction of wheels axis (The last defined as monotonic function of the abduction angles):

\[
Z_1 = m_1 g + \frac{m_2 g b_i (c - b)}{L_i}; \quad Z_2 = m_2 g a + \frac{m_2 g b_i (a + c)}{L_i}; \quad Z_3 = m_2 g d_i; \quad Y_i = \frac{k_i \delta_i}{\sqrt{1 + \gamma_i^2}}.
\]

In implementing rectilinear motion speed of SHZA undisturbed movement is defined as the ratio:

\[
v = \sqrt{\frac{\alpha Z_3 - kf(Z_1 + Z_2)}{K_{as}}},
\]  

where \( K_{as} \) – factor of streamlining of SHZA.
In the case of stable circular regime, value of longitudinal speed of movement of first link is determined by solution of the final equation with other equilibrium phase variables:

\[ m_0 \omega - k Z_2 - K_2 v - \sin \theta Y_1 - \cos \theta k_1 Z_1 + m_1 \omega \alpha + m_1 \alpha \omega^2 - m_1 \cos \theta \omega^2 + \alpha \cos \phi Z_3 + \\
+ \sin \phi Y_1 - m_2 \cos \phi \omega^2 + m_2 \mu \omega - m_3 \cos \omega^2 = 0 \\
+ m_2 \sin \phi \omega^2 + Y_2 - m_1 \omega - m_2 \omega^2 = 0; \\
+ m_2 \sin \phi \omega^2 + Y_2 - m_1 \omega - m_2 \omega^2 = 0; \\
\]

\[ \begin{align*}
   c m_2 \omega - a m_2 \omega - a \sin \theta k_1 Z_1 + c \sin (\phi + \theta) \alpha Z_3 + a \cos \theta Y_1 - c \cos (\phi + \theta) Y_3 - b Y_2 + \\
+ k k_2 (\theta - \theta_0) + k k_2 (\phi - \phi_0) - a m_1 \sin \theta \omega^2 - c m_2 \sin \phi \omega^2 = 0; \\
\end{align*} \]

\[ \lambda(Y_1 - \sin \theta m_1 \omega - \sin \theta m_1 \alpha \omega^2 - \cos \theta m_1 \omega) - k k_2 (\theta - \theta_0) = 0; \\
- L_1 Y_3 \cos \theta + L_0 \alpha Z_3 \sin \theta + d_1 \cos \phi m_2 \alpha - d_1 \sin \phi m_2 \alpha + c m_2 \sin \phi \omega^2 - k k_2 (\phi - \phi_0) = 0 \]  

(22)

(in the system that determines the equilibrium value, the phase velocity \( (\Theta, \Phi) \) is equal to zero).

Rectilinear motion of SHZA corresponds to zero value of parameter \( \theta_0 \). At \( \theta_0 \rightarrow 0 \) to trivial decision strive the circular steady state of sufficiently large radius. It enables to apply graph-analytical method for the extension for parameter by moving on branch of equilibrium states.

Below are values of phase variables corresponding to the equilibrium state of the system that can serve as a starting point in the implementation of the method of continuation by parameter (along with trivial mode).

Integrating the system of equations (22) carried on the same data source typical for SHZA.

The values of the phase variables that correspond to stationary states of SHZA and sets the corresponding values to it: \( \theta_0 = 0.1 \); \( X_3 = 0.123 \).

\[ \begin{align*}
   \{ u = -.1454343890e-1, v = 5.629191511, \omega = .1451613858, \phi = .1958011278, \theta_0 = .1019656310 \}; \\
   \end{align*} \]

eigenvalues of matrix coefficients of equations (22) indicate that the circular stationary regime is stable.

\[ \begin{align*}
   \text{eigv} := & -6.986921098, -3.969798706 - 5.108965081 I, -3.969798706 + 5.108965081 I, \\
& -1.057375428, -.9785077698 - 13.90994274 I, -.9785077698 + 13.90994274 I, \\
& -3.554712212e-1
\end{align*} \]

With an increase of traction effort in two times we have the next stationary mode:

\( \theta_0 = 0.1 \); \( X_3 = 0.2 Z_3 \).

\[ \begin{align*}
   \{ \omega = .3597611916, \phi = .4113165810, \theta_0 = .1054213101, u = -.8090095905, v = 7.435758605 \}; \\
   \end{align*} \]

eigenvalues of matrix coefficients of equations (22) indicate that the circular stationary regime is stable.

\[ \begin{align*}
   \text{eigv} := & -5.373724168, -2.185804771 - 1.873731709 I, -2.185804771 + 1.873731709 I, \\
& -1.052815285 - 13.90091946 I, -1.052815285 + 13.90091946 I, \\
& -9.928401012e-1 - .4632056434 I, -.9928401012e-1 + .4632056434 I
\end{align*} \]

At further increase of traction force, circular stationary regime loses stability:

\( \theta_0 = 0.1 \); \( X_3 = 0.3 Z_3 \).

\[ \begin{align*}
   \{ \phi = .6220454182, u = -1.136704740, \omega = .4735732903, v = 6.455904321, \theta_0 = .1024505047 \}; \\
   \end{align*} \]

eigenvalues of matrix coefficients of equations (22) indicate that the circular stationary regime is stable.

\[ \begin{align*}
   \text{eigv} := & -6.146575049, -2.136688971 - 1.662291704 I, -2.136688971 + 1.662291704 I, \\
& -1.009804871 - 13.87431469 I, -1.009804871 + 13.87431469 I, \\
& .1033177514 - .5562877329 I, .1033177514 + .5562877329 I
\end{align*} \]

At a set of eigenvalues indicates flatten instability corresponding to steady state, the system allowing the existence of another one more not stable circular mode, for which along with flatten instability takes place divergent imbalance:

\[ \begin{align*}
   \{ u = 4.774121575, \phi = .5688084995, \omega = 2.469381759, \theta_0 = .6912609845e-1, v = .1568547332 \}; \\
   \end{align*} \]

eigenvalues of matrix coefficients of equations (22) indicate that the circular stationary regime is stable.

\[ \begin{align*}
   \text{eigv} := & -2.441157656 - 2.744480930 I, -2.441157656 + 2.744480930 I, \\
& -1.226538298 - 14.66281452 I, -1.226538298 + 14.66281452 I, \\
& .2033799634 - 3.715976489 I, .2033799634 + 3.715976489 I, .2456426373
\end{align*} \]
In Fig. 1, Fig. 2 are shown the trajectory of the center of mass of the tractor SHZA in the plane of the road, phase trajectory angle of drafting and angle of rotation of CCM at different values of traction force that developing on the axis of the trailer.

Analysis of Fig. 1, Fig. 2 shows a loss of stability of circular stationary modes implemented at parameter values 0.5<alpha<0.75, and loss of stability of rectilinear motion at alpha = 0.245. The critical speed in this case is v_{kp} = 12.38 m/s.

Critical speed of steady circular motion essentially depends on the values of parameter theta0. Loss of stability occurs with the emergence of multiple equilibrium (due to a bifurcation or merge of birthed multiple stationary modes). According to the results of applied catastrophe’s theory many critical in some small neighborhood of rectilinear motion should be implemented in the form of semi cubic parabola.

Analysis of cumbersome conditions of static stability of SHZA at the implementation traction force on the axle of the trailer showed that value of coefficient of aerodynamic resistance will not affect the value of the critical velocity (not included in the expression of critical speed), and the coefficients values of resistance movement in the first and second axes slightly affect the value of critical speed and critical parameter alpha, which determines the amount of traction.

Analysis of Fig. 3 shows the significant dependence of the critical speed of SHZA movement of the coefficient value, which shows what part of vertical load on the axle of the trailer is realized in the form of traction.

In accordance with the accepted assumptions in this paper, each value of parameter alpha > alpha' corresponds to a certain value of sustainable longitudinal speed of movement

$$v = \sqrt{\frac{\alpha Z3 - kf(Z1 + Z2)}{Kas}}.$$  

Thus, the point of intersection of two dependencies (received before and this) determines the value of critical velocity and critical of parameter alpha, Fig.3 (c).

Analysis of the relationship, Fig. 3 (c) shows that even for optimum selected stiffness and layout parameters critical speed of SHZA is not more than 12.5 m/s, thus, such buses require special system versus compilation; however at movement with speed lower than values of critical speed of movement, SHZA stability is provided without additional versus compilation measures.

theta0=0.1 ; X3=0.75 Z3

![Diagram](image1)

![Diagram](image2)

![Diagram](image3)

![Diagram](image4)

Fig.1 - Parameters of SHZA’s stationary movement at the value of pushing force X3=0.75Z3
theta0=0.1 ; X3=0.2 Z3

Fig.2 - Parameters of SHZA’s stationary movement at the value of pushing force X3=0.2Z3

Fig. 3 - Critical velocity of SHZA: the stiffness coefficient of control wheeled module of trailers section: 

\( kk_1=4000 \text{ N/rad} \) (a), the value of traction effort on the axis of trailers link at \( kf=0.025 \) (b), parameter alpha (c)

CONCLUSIONS

In the article is determined the necessary power of electrical motor that was established on trailing section of the bus, taking into account the pick-up from the place, rectilinear motion and circular motion. In the work is shown that at power of electrical motor of 19 kW, bus can move at maximum speed of 4.4 m/s, which is less than the critical speed, therefore stability of the bus movement in this case, will be provided. However, even under optimum selected parameters of stiffness and layout, critical movement speed of the hinge-articulated buses in the implementation of traction on the axle of trailer’s section does not exceed 12.5 m/s, that is why such buses require special system of versus compilation.
REFERENCES


