COMPARATIVE ANALYSIS OF SOME TIRE DEFORMATION MODELS USED FOR THE PREDICTION OF TRACTION CHARACTERISTICS

ANALIZA COMPARATIVĂ A UNOR MODELE DE DEFORMARE A PNEULUI ÎN SCOPUL OBŢINERII CARACTERISTICILOR DE TRACŢIUNE


\(^1\) University of Agricultural Sciences and Veterinary Medicine “Ion Ionescu de la Brad” Iași / Romania;
Tel: 0232-407561; E-mail: roscar@uaiasi.ro

Keywords: tire traction, super-ellipse, traction force

ABSTRACT

In the present paper the deformation of the tire section is under load is taken into account. The tire section is considered to be elliptical; under load the minor axis decreases, while the major axis increases. The equations of the traction model were incorporated in a computer program; the length, width and area of the contact patch, the traction force and traction efficiency are calculated for each value of the wheel slip.

Field tests were performed in order to validate the model; the experimental data were collected during plowing tests. In order to evaluate the precision of the model the predicted data were compared with the test data by the means of a goodness-of-fit analysis.

INTRODUCTION

The tire-soil interaction models can be based on empirical, semi-empirical and analytical methods (Tiwari et al., 2010).

Empirical methods are mainly based on soil properties (cone index, plate sinkage, shear strength) using similitude and dimensional analysis. At the end of the Second World War, this approach evolved as a means of measuring trafficability of soil at the US Army Corps of Engineers, Waterways Experiment Station (Tiwari et al., 2010). The empirical models were developed using traction data recorded from operating vehicles; for some of them, cone index, measured with a standard cone penetrometer, was the only soil property taken into account. Wismer and Luth (1972) developed a widely used model for bias tyres, based on a soil-tyre numeric, which under-predicted the traction force when applied to radial tyres. The Brixius (1987) equations, as a refinement of the Wismer and Luth equations, expressed the gross traction ratio (GTR) as a function of slip and wheel mobility number, using a curve fitting technique in order to evaluate the coefficients for the traction equation (Lee et al., 2016).

The semi-empirical models represent a physical-based approach, which considers the mechanics of the soil under load is taken into account. The wheel-soil interaction and are suitable for practical applications (Battiato and Diserens, 2017). In the semi-empirical models, the shear deformation of soil is considered; the models are based on soil parameters obtained by the means of a bevameter technique (penetration and shear tests), assuming that the vertical deformation of soil is similar to the deformation under a sinking plate, while the shear deformation of soil under a traction device is similar to the shear action of a torsion device (Tiwari et al., 2010). The parameters involved in the equations are determined experimentally. For agricultural soils, the Janoshi and Hanamoto (1961) equation is one of the most frequently used.

The analytical models are formulated using elasticity and plasticity approaches. Elasticity models are based on the classical mechanical contact theory in order to predict deformations and stresses (using, for example, the Boussinesq’s approach), while plasticity based models take into account material (soil) failure...
theories. Despite the rather sophisticated theoretical base of these models, there are authors (Upadhya et al. 1990; Xia, 2011) who concluded that analytical models never adequately describe the interaction between tyre and soil due to the large number of soil parameters that should be taken into account and to their variability.

The tire-soil interaction models are used in order to predict the wheel traction force and traction efficiency. Besides these, they also take into account the shape and area of contact patch between tire and soil, which is also used for the calculation of the surface pressure and is also considered by the models for stress propagation in soil and for the prediction of the compaction risk (Diserens et al., 2011).

In a previous paper (Roşca et al., 2014) a semi-empirical model for predicting the traction force and traction efficiency was presented; the model was applied on a 2WD agricultural tractor, assuming that the shape of the tire-ground contact area is a super ellipse. The model assumed that the super ellipse equation describes the shape of the tire-ground contact surface and was considered to be a reasonable compromise between the more simple empirical models, for which the range of applicability is limited to the cases having similar conditions to the ones from which the models were derived, and the analytical models, which require in-situ evaluation of a large number of soil properties. Experimental results from plowing tests were used in order to validate and verify the applicability of the model, using a goodness-of-fit analysis. For the case of the traction force, the value of the Pearson r² correlation coefficient achieved values between 0.921 and 0.925, thus confirming the validity of the model; for the case of traction efficiency lower values of the Pearson coefficient were obtained, due to the lower values predicted by the model at wheel slips below 15%.

One of the key elements of the above mentioned model was the tire change in volume due to deflection under load, which was calculated considering that the tire radius increases as the tire flattens in the contact area; the tire width was considered constant.

In order to improve the theoretical results regarding the wheel traction efficiency in the present paper the deformation of the tire section was also taken into account. The tire section was considered to be elliptical; under the vertical load the minor axis decreased, while the major axis (tire width) increased.

MATERIAL AND METHODS

The tire-soil interaction model is based on the one developed earlier (Roşca R. et al., 2014) and its schematics is shown in fig. 1: under the vertical load (G), the wheel sinks into the soil, reaching the depth (zc) and the load induces tyre deflection (zp); as a result, the radius of the contact patch becomes rd (rd>r0), and the length of the contact patch is:

\[ I_c = 2 r_d \sin \beta = 2 r_0 \sin \alpha. \] (1)

Fig. 1 – Schematics of the wheel-soil interaction model

The shape of the contact patch is assumed to be a super ellipse (Keller, 2005):

\[ \left( \frac{2 \cdot x}{I_c} \right)^k + \left( \frac{2 \cdot y}{I_w} \right)^k = 1, \] (2)
where \( k \) is the super ellipse exponent, the minor axis of the super ellipse is assumed to be equal to the tyre width \( b \) (Keller, 2005). \( l_c \) is the major axis of the super ellipse (length of the contact area) and \( l_w \) is the minor axis of the super ellipse (width of the contact area).

The tire-soil pressure was defined using the pressure-sinkage relationship:

\[
p = k \cdot z^n,
\]

where \( p \) is the normal pressure [kPa], \( z \) is the deformation [m], and \( k \) [kPa/m\(^n\)] and \( n \) are constants.

Based on the tire-soil pressure and assuming that the tire is perfectly elastic (Ghiulai and Vasiliu, 1975) finally leads to:

\[
G = \int_0^{2\pi} p \cdot b(\varphi) \cdot r_d \cdot \cos(\beta - \varphi) \cdot d\varphi = q_p \cdot \Delta V_p,
\]

where \( \varphi \) is the current angle, defining the position along the contact surface, \( p \) is the normal pressure, \( q_p \) is the tire volume stiffness and \( \Delta V_p \) is the tire change in volume due to deflection.

In the initial paper (Roşca et. al., 2014) the tire change in volume due to deflection \( \Delta V_p \) was evaluated considering that the tire radius increases from \( r_0 \) to \( r_d \) as the tire flattens in the contact area, while the tire width was considered constant (fig. 2).

Finally, after several transformations of equation (4), the following equation is obtained:

\[
k \cdot \int_0^{2\pi} b(\varphi) \cdot r_d^{n+1} \cdot [\cos(\beta - \varphi) - \cos \beta]^n \cdot \cos(\beta - \varphi) \cdot d\varphi + \frac{4}{3} b \cdot q_p \cdot \beta^3 \cdot r_d^2 = \frac{4}{3} b \cdot q_p \cdot \alpha^3 \cdot r_0^2.
\]

In the present study the deformation of the tire cross-section is considered; the shape of the tire cross-section is approximated by an ellipse (Koutný, 2007), as shown in fig. 3a. Under the effect of vertical load (G, fig. 1), the cross-section is deformed, but the elliptical shape is preserved (fig. 3b): the minor semi-axis decreases and becomes \( h-z_p \) while the major axis increases from \( b \) (tire width in the unloaded condition) to \( l_w \).
The major axis of the ellipse is calculated assuming that its perimeter remains constant:

\[
2\pi \cdot \sqrt{\frac{(b/2)^2 + (h/2)^2}{2}} = 2\pi \cdot \sqrt{\frac{(l_w/2)^2 + (h-z_p)/2}{2}},
\]

which results in:

\[
l_w = \sqrt{b^2 + 2 \cdot h \cdot z_p - z_p^2}.
\]

The tire volume change \( \Delta V_p \) is calculated as the cross-section area multiplied by the length of the contact patch, resulting in:

\[
\Delta V_p = 2 \cdot \alpha \cdot r_o \cdot \pi \cdot \frac{b}{2} \frac{h}{2} - 2 \cdot \beta \cdot r_d \cdot \pi \cdot \frac{l_w}{2} \frac{h-z_p}{2} = 0.5 \cdot \pi \cdot [\alpha \cdot r_o \cdot b \cdot h - \beta \cdot r_d \cdot l_w \cdot (h-z_p)].
\]

After several transformations, equation (4) becomes:

\[
k \cdot \int_0^{2\pi} b(\phi) \cdot r_d^{n+1} \cdot [\cos(\beta - \phi) - \cos \beta]^n \cdot \cos(\beta - \phi) \cdot d\phi = q_p \cdot 0.5 \cdot \pi \cdot [\alpha \cdot r_o \cdot b \cdot h - \beta \cdot r_d \cdot l_w \cdot (h-z_p)]
\]

The following equations are also obtained from fig. 1:

\[
z_c = r_o - z_p - r_o \cdot \cos \alpha,
\]

\[
z_p = r_o \cdot (1 - \cos \alpha) - r_d \cdot (1 - \cos \beta),
\]

where \( z_p \) is tyre deflection under vertical load.

A computer program is used in order to solve the systems of equations (1), (5), (10), (11) and (1), (9), (10), (11), respectively. The program displays the following values (fig. 4):

- length of the contact patch, \( l_c \);
- width of the contact patch, \( l_w \);
- area of the contact patch;
- calculated value of tire deformation under load, \( z_p \);
- tire sinkage, \( z_p \);
- dynamic radius of the wheel, \( r_d \);
- angle of the contact patch, \( \beta \);
- maximum shear stress, \( \tau_{\text{max}} \);
- normal pressure, \( p \).

The values of slip, traction force and traction efficiency are saved in a file.

Fig. 4 – Output screen of the computer program
In order to evaluate the maximum traction force it is assumed that it is limited only by the soil shear strength; the Mohr-Coulomb equation is used to calculate the soil maximum shear stress:
\[ \tau_{\text{max}} = c + p \cdot \tan \gamma, \]
where \( c \) is soil cohesion [kPa], \( p \) is the vertical pressure [kPa] and \( \gamma \) is the internal friction angle.

Soil shear tension was calculated using the Janosi and Hanamoto (1961) equation:
\[ \tau = \tau_{\text{max}} \cdot \left( 1 - \frac{J}{K} \right), \]
where \( K \) is the soil shear deformation modulus and \( J \) is the shear displacement.

The maximum traction force was calculated as the product of shear stress and shear area; according to ASAE S296, the net traction force is \( F_N = F_t - R_r \), with the wheel rolling resistance \( R_r \) being calculated with the relation (Elwaleed et al., 2006):
\[ R_r = \frac{G \cdot 1}{B_n} \left( 1 + 0.04 + \frac{0.5 \cdot s}{\sqrt{B_n}} \right) \text{[kN]}. \]

The wheel numeric \( B_n \) is (ASAE D497.7, 1999):
\[ B_n = \frac{C_l \cdot b \cdot d + 1 + 5 \cdot \frac{z_n}{h} + 1 + 3 \cdot \frac{b}{d}}{G}, \]
where \( C_l \) is the soil cone index [kPa], \( d = 2r_0 \text{[m]} \) and \( h \) is the tyre section height [m].

The same standard defines the traction efficiency as:
\[ \eta_{tr} = (1 - s) \cdot (1 - R_r / F_t). \]

The model was applied to the driving wheel of an U-650 agricultural tractor; the main characteristics of the wheel are presented in table 1.

<table>
<thead>
<tr>
<th>Item</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load on the driving tire, ( G ) [kN]</td>
<td>11.75</td>
</tr>
<tr>
<td>Type of tire</td>
<td>14.00-38</td>
</tr>
<tr>
<td>Rim diameter, ( d_i ) [m]</td>
<td>0.965</td>
</tr>
<tr>
<td>Section height, ( h ) [m]</td>
<td>0.307</td>
</tr>
<tr>
<td>Exterior diameter of tire, ( d_i + 2 \cdot h ) [m]</td>
<td>1.58</td>
</tr>
<tr>
<td>Tire width, ( b ) [m]</td>
<td>0.370</td>
</tr>
<tr>
<td>Tire inflation pressure [kPa]</td>
<td>100</td>
</tr>
</tbody>
</table>

The experimental data were collected during the field tests performed with the U650+P2V plowing unit; the tractor was fitted with a dynamometric frame and an electronic dynamometer, as shown in fig. 5. The plough was mounted on the dynamometric frame and thus the net traction force \( F_{t,ef,r} \) was measured directly. During these tests drive wheel slip did not exceed 30% because of the restraints imposed by the plowing process. Different traction forces and drive wheel slips were achieved by changing the operating width and depth of the plough. The traction force for each experimental point was calculated as the average value of nine measurements; the standard error and 95% confidence interval were then evaluated. The experimental results taken into account for the goodness-of-fit analysis are shown in table 2.
Table 2

<table>
<thead>
<tr>
<th>Wheel slip [%]</th>
<th>Average traction force [kN]</th>
<th>Standard error</th>
<th>95% data confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>1.8033</td>
<td>0.095</td>
<td>0.2179</td>
</tr>
<tr>
<td>9</td>
<td>2.3744</td>
<td>0.116</td>
<td>0.2684</td>
</tr>
<tr>
<td>14</td>
<td>3.1033</td>
<td>0.082</td>
<td>0.1891</td>
</tr>
<tr>
<td>17</td>
<td>3.3067</td>
<td>0.121</td>
<td>0.2797</td>
</tr>
<tr>
<td>18</td>
<td>3.6855</td>
<td>0.116</td>
<td>0.2680</td>
</tr>
<tr>
<td>20</td>
<td>4.3122</td>
<td>0.175</td>
<td>0.4029</td>
</tr>
<tr>
<td>25</td>
<td>4.6777</td>
<td>0.087</td>
<td>0.2003</td>
</tr>
<tr>
<td>26</td>
<td>4.7567</td>
<td>0.114</td>
<td>0.2636</td>
</tr>
<tr>
<td>29</td>
<td>5.8800</td>
<td>0.141</td>
<td>0.3249</td>
</tr>
</tbody>
</table>

Table 3

<table>
<thead>
<tr>
<th>Characteristics of the test soil</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item</td>
</tr>
<tr>
<td>Soil type</td>
</tr>
<tr>
<td>Average bulk density [kg/m^3]</td>
</tr>
<tr>
<td>Average soil water content [%]</td>
</tr>
<tr>
<td>Soil deformation modulus, K [m]</td>
</tr>
<tr>
<td>Coefficients for the pressure-</td>
</tr>
<tr>
<td>sinkage equation</td>
</tr>
<tr>
<td>k</td>
</tr>
<tr>
<td>n</td>
</tr>
<tr>
<td>Soil cohesion, c [kPa]</td>
</tr>
<tr>
<td>Angle of internal friction, γ [°]</td>
</tr>
<tr>
<td>Cone penetrometer index, C [kPa]</td>
</tr>
</tbody>
</table>

The tests were performed on wheat stubble sandy loam soil, after cereal harvesting; soil characteristics in the test field are presented in table 3.

Table 4 presents some comparative results given by the tire – soil

RESULTS

For the both tire section deformation models the calculations were performed using the same value for the super ellipse coefficient (k=3.5). Table 4 presents some comparative results given by the tire – soil
computer simulation; the assumption that the tire cross-section has an elliptical shape and is deformed due to the vertical load of the tire had the following consequences:

- while the length of the contact patch decreased slightly (from 0.533 m to 0.531 m), the width of the contact patch, \( l_w \), increased from 0.3 m to 0.319 m, resulting in a larger area of the contact surface (0.154 m\(^2\));
- the tire radius \( r_d \) decreased from 1.4 m to 1.371 m; as the length of the contact patch did not change significantly, the centre angle \( \beta \) increased from 10.916° to 11.19°;
- the maximum shear stress decreased from 53.3 kPa to 52.01 kPa due to the increase of the contact surface area.

### Table 4

<table>
<thead>
<tr>
<th>Item</th>
<th>Initial tire deformation model (b const.)</th>
<th>Modified tire deformation model (b ellipse)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of the contact patch, ( l_c ) [m]</td>
<td>0.533</td>
<td>0.531</td>
</tr>
<tr>
<td>Width of the contact patch, ( l_w ) [m]</td>
<td>0.300</td>
<td>0.319</td>
</tr>
<tr>
<td>Tire deflection, ( z_d ) [m]</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>Sinkage depth, ( z_c ) [m]</td>
<td>0.027</td>
<td>0.027</td>
</tr>
<tr>
<td>Area of the contact surface, ( A_t ) [m(^2)]</td>
<td>0.145</td>
<td>0.154</td>
</tr>
<tr>
<td>Tire radius, ( r_d ) [m]</td>
<td>1.400</td>
<td>1.371</td>
</tr>
<tr>
<td>Centre angle of the contact patch, ( \beta ) [°]</td>
<td>10.916</td>
<td>11.19</td>
</tr>
<tr>
<td>Maximum shear stress, ( \tau_{\text{max}} ) [kPa]</td>
<td>53.3</td>
<td>52.01</td>
</tr>
</tbody>
</table>

Fig. 6 and 7 present the predicted and experimental results concerning the traction force and traction efficiency. The charts clearly show that the model predicted higher values of the traction force and traction efficiency when the deformation of the tire cross section was considered, due to the increased value of the contact surface area.

The results of the goodness-of-fit analysis are shown in table 5. Compared to the previous model, the most significant differences were recorded for the traction efficiency: the Pearson correlation coefficient \( r^2 \) increased from 0.186 to 0.216, the mean absolute deviation (MAD) decreased from 0.058 to 0.051, root mean squared deviation (RMSD) decreased from 0.0752 to 0.0686 and the mean scaled absolute deviation (MSAD) decreased from 5.225 to 4.557.

### Table 5

<table>
<thead>
<tr>
<th>Item</th>
<th>Traction force</th>
<th>Traction efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Constant cross-section</td>
<td>Deformable cross-section</td>
</tr>
<tr>
<td>( r^2 )</td>
<td>0.923</td>
<td>0.924</td>
</tr>
<tr>
<td>PW95CI</td>
<td>66.7</td>
<td>55.6</td>
</tr>
<tr>
<td>MAD</td>
<td>0.354</td>
<td>0.356</td>
</tr>
<tr>
<td>RMSD</td>
<td>0.480</td>
<td>0.438</td>
</tr>
<tr>
<td>MSAD</td>
<td>3.122</td>
<td>3.065</td>
</tr>
</tbody>
</table>
When referring to the values of the traction force, all the goodness-of-fit parameters recorded better values for the modified traction model, excepting the percentage of points within 95% confidence interval of data (Pw95CI), which has slightly decreased (from 66.7% to 55.6 %).

CONCLUSIONS

A modified semi-empirical model for the prediction of traction performance of a tractor driving wheel is presented in this study. The model assumed that the super ellipse equation describes the shape of the tire-ground contact surface.

The model is a reasonable compromise between the more simple empirical models, for which the range of applicability is limited to the cases having similar conditions to the ones from which the models were derived, and the analytical models, which require in-situ evaluation of a large number of soil properties.

Experimental results from plowing tests were used in order to validate and verify the applicability of the model, by the means of a goodness-of-fit analysis. The analysis showed that the modified traction model provided more accurate results regarding the traction force and traction efficiency than the initial one.

REFERENCES